TPC Desktop Series

Least Squares Learning Guide

1/18

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Typography

We will be using the following typography to distinguish items in the text:

[Enter] Brackets indicate keys on the keyboard.

flexibility Italics are used to highlight words for more emphasis.

Lot 2 Numbers or text that you enter.

System Requirements

This version of TPC Desktop requires Windows 10 / 8.1 / 8 / 7 / Vista and Internet Explorer Version 6.0 or later. We recommend you have 2 GB of RAM 32-bit OS) 4 GB RAM (64-bit OS) and 60 MB of free hard disk space in which to install the program. **An internet connection is required for program validation.**

Important Licensing Information

Our License Agreement allows you to use TPC on only one computer at a time unless you purchase additional licenses. We do offer one courtesy installation for an additional home or field computer, *but not for a second user* and assuming that the two programs are not being used at the same time. To request a courtesy license contact Traverse PC.
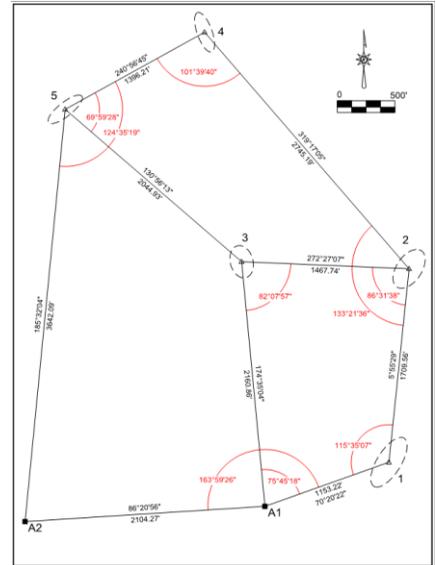
Least Squares Network Adjustment

▶**This chapter demonstrates:**

➢ Entering redundant observations

➢ Creating 'Fixed' coordinates

➢ Setting the A Priori values

➢ Analyzing a survey (creating a network)

➢ Editing network points

➢ Editing network observations

➢ Solving the network

➢ Creating the least squares report

➢ Determining the strength of your solution

➢ Adjusting the survey

➢ Plotting error ellipses

Overview

In this example, you'll work through a small Least Squares Network that includes two traverses and an extra observation. The principles are the same for larger networks involving many traverses and possibly hundreds of unknown points and observations.

▶**Professional Edition**

The Least Squares Network analysis is available only in the Professional edition of TPC. Least Squares Traverse analysis (not covered here) is available from the Closure View in both the Premium and Professional editions.

✍ **To learn more about Least Squares**

1. Choose **Help | Search For Help On…** and left-click the **Contents** tab.

2. Scroll down to **Least Squares**.

3. Double-click **Least Squares** to expand the chapter and then double-click **Least Squares Overview**.

▶**Getting Started**

1. From the Tasks Manager choose **Sample Surveys**.

2. Double-click the **LEARN LEAST SQUARES NETWORK.TRV** file.

3. Choose **File | Save Survey As** then enter the name **Least Squares Network** and choose **Save**. If prompted to replace the existing file, choose **Yes**.

Least Squares Network Adjustment
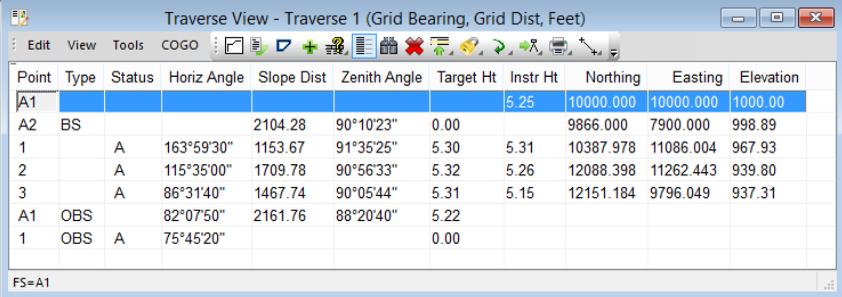
Entering Redundant Observations

At the heart of any Least Squares solution are redundant observations – measuring the same point more than once, from different control points, in order to provide a statistical basis for the computed positions of the unknown points.

1. From the Traverses Manager, double-click **Traverse 1** to open its Traverse View.

Points A1 and 1 at the end of the traverse are redundant observations. The **OBS** in the **Type** column indicates that these are Observations (measured data is stored but the points are not recomputed).

Let's delete these two points and re-enter them.

Point	Type	Status	Horiz Angle	Slope Dist	Zenith Angle	Target Ht	Instr Ht	Northing	Easting	Elevation
A1							5.25	10000.000	10000.000	1000.00
A2	BS			2104.28	90°10'23"	0.00		9866.000	7900.000	998.89
1		A	163°59'30"	1153.67	91°35'25"	5.30	5.31	10387.978	11086.004	967.93
2		A	115°35'00"	1709.78	90°56'33"	5.32	5.26	12088.398	11262.443	939.80
3		A	86°31'40"	1467.74	90°05'44"	5.31	5.15	12151.184	9796.049	937.31
A1	OBS		82°07'50"	2161.76	88°20'40"	5.22				
1	OBS	A	75°45'20"			0.00				

Traverse View - Traverse 1 (Grid Bearing, Grid Dist, Feet)

Edit   View   Tools   COGO

FS=A1

2. Select points A1 and 1 at the end of the traverse, choose **Edit | Delete Selected Items** and then choose **Yes**.
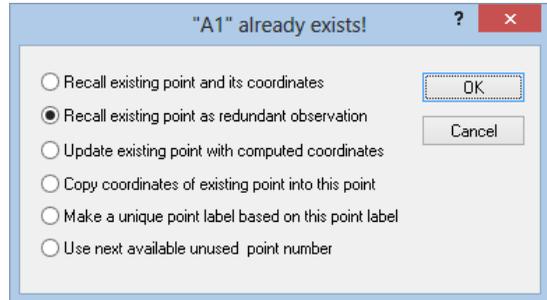
3. Press the down arrow key to create a new point below point **3**. TPC will create the next available point number like 6 or 7.

4. Highlight the point column of the added point, type **A1** and press [Enter]. TPC displays the **"A1" already exists!** dialog.

5. Choose the second option **Recall existing point as redundant observation** and choose **OK**.

6. TPC replaces the point in the Traverse View with the recalled **A1** point and sets its point **Type** to **OBS**.

7. Enter the **Horiz Angle**, **Slope Dist**, **Zenith Angle** and **Target Ht** shown above for point A1.

"A1" already exists!

○ Recall existing point and its coordinates
● Recall existing point as redundant observation
○ Update existing point with computed coordinates
○ Copy coordinates of existing point into this point
○ Make a unique point label based on this point label
○ Use next available unused point number

OK

Cancel

8. Repeat steps 3 – 7, recalling point 1 as a redundant observation and entering just the **Horiz Angle** shown above.

Congratulations! You've just entered redundant data.

✎See the **Redundant Data** help topic for other ways of entering redundant data.

Creating 'Fixed' Coordinates

In a Least Squares Analysis, at least two points must have *fixed* or *known* positions. When the network is solved, the positions of these fixed points are not allowed to change. In reality, you can have any number of fixed points in a Least Squares Network, a situation not at all uncommon as you build onto an existing network with new data.

Fortunately, TPC has long had the ability to *protect* points. The positions of *protected* points are not allowed to change – just like the *fixed* points in a Least Squares Network. Because of this strong correlation between *protected* and *fixed* points, TPC treats any *protected* point in the survey as a *fixed* point when it builds the Least Squares Network.

In the example data we are using, points A1 and A2 are the two points we want to fix for our Least Squares analysis. So we want to protect them in TPC before creating the network.

1. Highlight points A1 and A2 and choose **Edit | Modify Points | Protect On**. TPC displays the **Protection Options** dialog.

2. Choose **All Data** and click **OK**.

TPC displays a **PE** in the Status column of points A1 and A2. The **P** indicates the X and Y coordinates are protected.
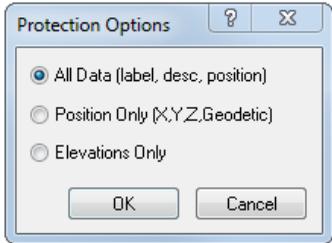
Point	Type	Status	Horiz Angle	Slope Dist	Zenith Angle	Target Ht	Instr Ht
A1		PE					5.25
A2	BS	PE		2104.28	90°10'23"	0.00	
1		A	163°59'30"	1153.67	91°35'25"	5.30	5.31
2		A	115°35'00"	1709.78	90°56'33"	5.32	5.26
3		A	86°31'40"	1467.74	90°05'44"	5.31	5.15
A1	OBS	PE	82°07'50"	2161.76	88°20'40"	5.22	
1	OBS	A	75°45'20"			0.00	

OP=A1 FS=A2

That's all there is to it. Later on, when we build the network, we will note that points A1 and A2 are automatically fixed.

►Making Least Squares Easier

TPC is a natural when it comes to building Least Squares Networks. The use of traverses to build onto an existing network and the ability to fix protected points are just two of the built-in features that allow TPC to simplify your use of Least Squares Networks.

Setting the A Priori Values

A Priori values are estimates of standard error for each type of measurement. TPC uses these to create the initial precision of each value used in the Least Squares Network. You can edit these precisions for each observation after TPC creates the network, but specifying correct values for the instrument you are using will reduce the editing of precisions later on.

1. From the desktop menu choose **Tools | Least Squares Network Adjustment**. TPC displays the **Least Squares Network Adjustment** dialog.

2. Left-click the **Miscellaneous** tab and choose **A Priori Values**.

3. Enter the values shown here.

4. Choose **Save Settings**.

You don't have to save these as your defaults now, but at some point, you will have your own estimates and will want to save them as the defaults.

5. Choose **OK** to close this dialog.

The values you enter here are saved in the Least Squares file, so if you don't choose to save them as the defaults (available for the next survey that uses Least Squares) then are still stored as part of this specific analysis.

▶Note: This sets the default Standard Errors (A Priori) values for the entire file. You can also set the A Priori values for individual traverses and over-ride the default A Priori values. You can do this either in the Traverses Manager and apply the settings to your selected traverses or in individual Traverse Views. In either case, choose **Tools | Least Squares Standard Errors** to set the A Priori values for individual or selected traverses.

▶Note: The default Network Least Squares Standard Errors (A Priori) values do not over-ride the individual traverse A Priori values. They only update traverses that have not had their A Priori values set.

Analyzing a Survey (Creating a Network)

A Least Squares solution in TPC involves four steps as follows:

1. Analyze the survey and create the network records
2. Solve the network and generate the appropriate statistics
3. Report the information about the solution so you can decide whether or not to accept it
4. Update the survey with the adjusted positions

In the analysis step, you tell TPC which traverses to use in building the network. TPC then looks at the points and measurements in these traverses to create the Least Squares Network.

To analyze this example survey:

5. Left-click the **Step 1 – Analyze** tab in the Least Squares Network Adjustment dialog.
6. Choose the **Analyze** button. TPC builds the network and displays the number of points and observations it found.

Notice that we choose the option to **Fix Protected Points**. The two points you protected earlier, A1 and A2, become the two fixed points for our network.

We also choose to include **All** the traverses in the survey, but we could have built a network using just the traverses that were selected in the Traverses Manager. This is how you add new data to an existing Least Squares Network.

For this example survey, TPC has determined that there are 5 unknown points, two known points and 18 observations, 8 of which are redundant. We'll look at these points and observations in the next task.
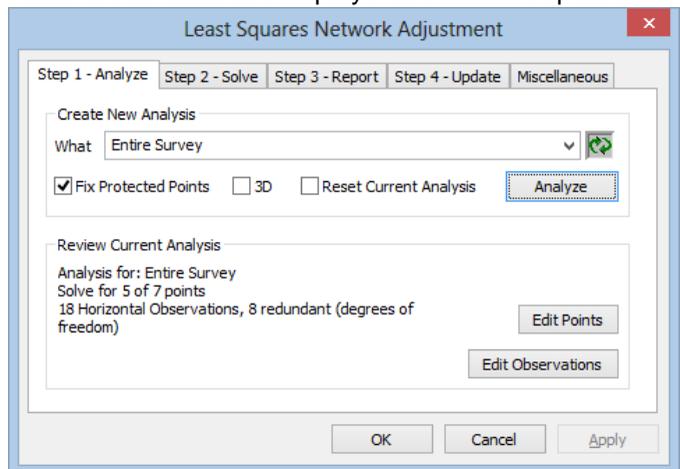
Editing Network Points

As a rule, you will not need to edit the points and observations TPC includes in the network. With the ability to group observations into traverses and protect survey points, TPC has all the tools it needs to build an accurate and complete network for you.

Let's verify this by taking a look at the points and observations you just created for this analysis.

1. From the **Step 1 – Analyze** page, choose **Edit Points**. TPC displays the **Least Squares Network Points** dialog.
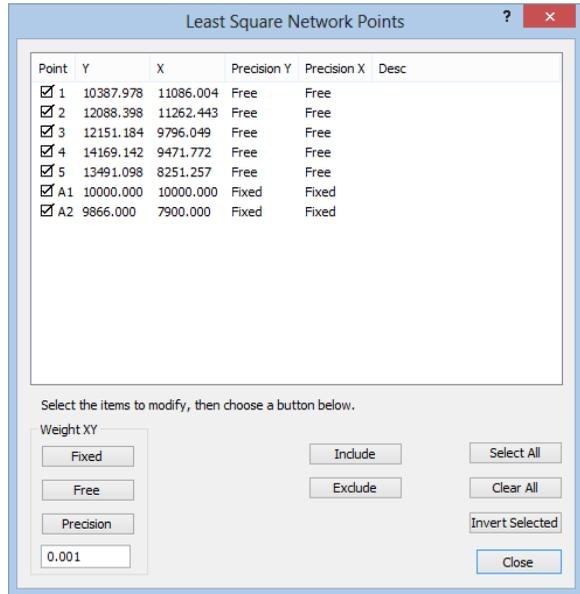
Notice that the **Precision Y** and **Precision X** for points A1 and **A2** is **Fixed**. These are the two points you protected earlier.

The **Precision** for the rest of the points is **Free**. This indicates that these points are free to be adjusted by any amount necessary for the solution of the network.

You can edit the **Precision** of any survey point in this dialog in order to limit the amount it gets adjusted.

You can also exclude points from the network. The checked box in the **Point** column indicates which points will be included in the solution. If a point is excluded, its position will not be affected by the adjustment.

Point	Y	X	Precision Y	Precision X	Desc
☑ 1	10387.978	11086.004	Free	Free	
☑ 2	12088.398	11262.443	Free	Free	
☑ 3	12151.184	9796.049	Free	Free	
☑ 4	14169.142	9471.772	Free	Free	
☑ 5	13491.098	8251.257	Free	Free	
☑ A1	10000.000	10000.000	Fixed	Fixed	
☑ A2	9866.000	7900.000	Fixed	Fixed	

Select the items to modify, then choose a button below.

Weight XY

Fixed	Include	Select All
Free	Exclude	Clear All
Precision		Invert Selected
0.001		Close

2. Choose **Close** to return to the **Step 1 – Analyze** tab.

Editing Network Observations

1. Choose **Edit Observations** to display the **Least Squares Network Observations** dialog. TPC displays each measurement along with its **Precision**.

The Precision values displayed here were derived from what are called *A PRIORI* values. These are estimates of the standard error of each type of measurement. Horizontal angles have their own estimate as do horizontal distances. You edited these estimates earlier in the A Priori dialog. TPC uses these estimates to establish the initial Precision of each measurement. You can edit the Precisions here if you choose to.

Notice that the slope distance and zenith angle values you entered in the Traverse View have been reduced to horizontal distance. In a 3D analysis, you will also see values for vertical distance. TPC uses the adjustments of each traverse, like Curvature and Refraction and Scale Factor to derive the horizontal and vertical values shown here.

Type	BS	OP	FS	Value	Precision
☑ Horiz Dist		A1	1	1153.23	0.056
☑ Horiz Angle	A2	A1	1	163°59'30"	10.000"
☑ Horiz Dist		1	2	1709.55	0.059
☑ Horiz Angle	A1	1	2	115°35'00"	10.000"
☑ Horiz Dist		2	3	1467.74	0.057
☑ Horiz Angle	1	2	3	86°31'40"	10.000"
☑ Horiz Dist		3	A1	2160.85	0.061
☑ Horiz Angle	2	3	A1	82°07'50"	10.000"
☑ Horiz Angle	3	A1	1	75°45'20"	10.000"
☑ Horiz Dist		2	4	2745.18	0.064
☑ Horiz Angle	1	2	4	133°21'40"	10.000"
☑ Horiz Dist		4	5	1396.21	0.057
☑ Horiz Angle	2	4	5	101°39'40"	10.000"
☑ Horiz Dist		5	A2	3642.09	0.068
☑ Horiz Angle	4	5	A2	124°35'20"	10.000"
☑ Horiz Angle	5	A2	A1	80°48'50"	10.000"
☑ Horiz Dist		5	3	2044.95	0.060
☑ Horiz Angle	4	5	3	69°59'30"	10.000"

Select the items to modify, then choose a button below.   [Select All]   ☐ Sort

[Precision]   [Include]   [Clear All]

[0.05]   [Exclude]   [Invert Selected]   [Close]

By reducing the raw data this way, the observed data used for the Least Squares network matches the initial coordinates for the unknown points.

2. When you are finished, choose **Close**.

Solving the Network

1. Left-click the **Step 2 – Solve** tab in the Least **Squares Network Adjustment** dialog.

2. Choose **Solve**.

TPC solves the network and indicates that the solution converged in 2 iterations. TPC also tells you that the solution 'Failed Horizontal Chi-Square test at the 95% significance level'.

▶ Note: If your solution does not converge go back and check your data selection.

You probably have a couple of questions about now like, "What are these iterations?" and, "What is this Chi-Square test?"

▶ What are Iterations?

A Least Squares solution is iterative – that means it needs to make one or more passes through the solution, hopefully improving the results with each pass until the results are acceptable. That's why you see the **Converge Limit** displayed in this page.

The Converge Limit says 'when the change in coordinates of all the unknown points from one pass to the next varies by less than 0.01 units, our solution is good and we can stop'.

The **Max Iterations** says to stop after the fifth pass, even if the solution has not converged.

▶ What is the Chi-Square Test?

The Chi-Square test compares the standard deviations of the unknown coordinates (how much they changed) with their initial positions, and asks if we are 95% sure that the amount of change is OK. This test fails when we have poor initial positions, bad observations or a very small data set (as in this example).

▶ That's All it Takes to Solve the Network?

Yes, that's all it takes. Larger networks will display a progress bar next to the **Solve** button so you can see the progress TPC is making as it iterates through the solution.
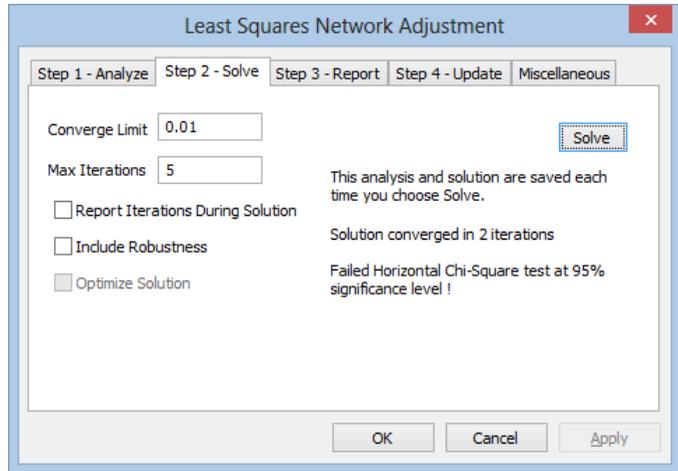
Creating the Least Squares Report

The Least Squares report provides statistical information about the solution. You will need this information to either accept or reject the solution.

1. Left-click the **Step 3 – Report** tab and choose the settings shown here. Make sure you have the **Residual / Corrected** option turned on.

2. Choose **Report**. TPC opens a **Least Squares Report** window and reports the results of the current solution.

3. Close the Least Squares Network Adjustment dialog by choosing **OK**.

4. Left-click anywhere on the Least Squares Report window to active it.

At this point, TPC has not adjusted any points in your survey. You are still just evaluating the Least Squares solution. You will look at the information in the report and determine the strength of the solution. You can also correct any blunders in the observed data, change the A Priori values or the Convergence Limit or whatever else you feel is appropriate in order to improve the solution.

So let's take a look at some of the information provided in the report.
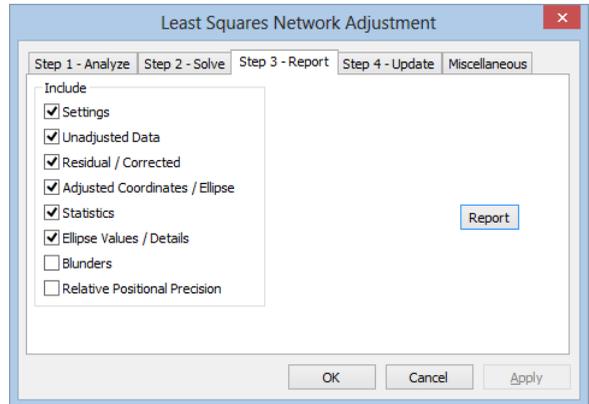
Determining the Strength of Your Solution

Locate the following sections in the report. You'll find information about the Chi-Square test and the Standard Deviation of Unit Weight in TPC's help topics under Least Squares or in most surveying texts that cover Least Squares adjustments. We'll focus on the Standard Deviation of Coordinates for our discussion here.

```
[[ Least Squares Solution ]]
Analysis for: Entire Survey
Solve for 5 of 7 points
18 Horizontal Observations, 8 redundant (degrees of freedom)
Solution converged in 2 iterations

95% Confidence F-statistic Std. Error Multiplier for 8 D.F. = 2.9863

[ Standard Error of Unit Weight ]
Observation      Count  StdDev
Horizontal         18   0.243


[[ Chi-Square ]]
Failed Horizontal Chi-Square test at 95% significance level !
Horizontal Chi-Square = 0.47
Lower/Upper Bounds (2.730/15.510)

Alternate Horizontal Chi-Square = 0.06 (divide by D.F.)
Lower/Upper Bounds (0.341/1.939)

[ Adjusted Coordinates ]
F-Statistic Multiplier (95% Confidence)= 2.986
Su=Semi-Major, Sv=Semi-Minor, t=Theta=angle from north (+CW, -CCW)

                     Adjusted Coordinates   Std Errors Error Ellipse
Station                     Y            X   Sy     Sx    Su     Sv           t
1                  10387.978    11086.001  0.032  0.034  0.034  0.032   30°06'12"
2                  12088.394    11262.451  0.048  0.057  0.066  0.035  -53°36'12"
3                  12151.204     9796.060  0.034  0.062  0.061  0.035  -83°39'00"
4                  14169.157     9471.786  0.040  0.094  0.094  0.040  -88°49'36"
5                  13491.116     8251.272  0.041  0.076  0.072  0.048  -77°10'16"
```

▶The Standard Deviation of Coordinates

When a standard deviation of a coordinate is expressed in terms of X and Y, it means that if you were to observe that point many times and compute a unique position each time, two thirds of those unique computed positions would lie within distance X and Y of the position being reported. So a smaller standard deviation indicates a stronger or more accurate position while a larger standard deviation indicates a weaker or less accurate position.

The deviations shown here indicate that our adjusted positions are within about six hundredths of where they would be if we repeated our observations many more times.

Plotting Error Ellipses

Plotting error ellipses helps you visualize the standard error of each computed point.

1. Left-click the **Miscellaneous** tab in the **Least Squares Network Adjustment** dialog.

2. Turn on the **Plot Ellipses** option and close the dialog by choosing **OK**.

3. Activate the Drawing View by left-clicking on it then choose **View | Regenerate**. You should see an error ellipse around each adjusted point.

▶Note: Error ellipses can only be plotted during the session in which you solve the least squares network and before the adjustment is applied. If you open this survey in another session, TPC will not draw the error ellipses.

▶Note: The interior angles were drawn in the picture shown here with the **Insert | Interior Angle** command in Drawing View.
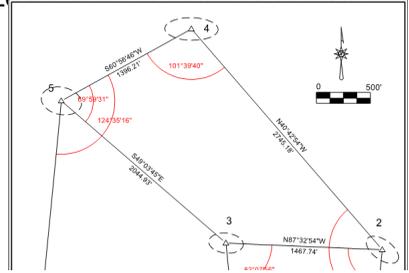
Adjusting the Survey

The final step is to accept the adjustments and update the survey.

1. From the desktop menu choose **Tools | Least Squares Network Adjustment**.

2. Left-click the **Step 4 – Update** tab.

3. Select the options shown here.

4. Choose **Update**.

TPC copies the adjusted coordinates to their respective points in the survey. Because you chose to **Protect Computed Points** TPC will also protect the adjusted points.

▶The Least Squares File

TPC stores all of the information about the network in a Least Squares file for this survey. This file contains the original (unadjusted) positions for each network point, the raw data for each observation, the A Priori values used in the analysis and other network information.

If you were to close this survey, then open it again and choose **Tools | Least Squares Network Adjustment**, TPC would open the Least Squares file and recreate this analysis. You would then **Solve** the network to recreate the solution (which is not stored in the Least Squares file since it can just be recomputed).

▶Choosing to Undo an Adjustment

You can always undo a Least Squares adjustment by returning here and choosing **Undo**. TPC copies the original coordinates back to each adjusted point and un-protects any points protected by the adjustment.